MANIFOLDS AND GROUPS IN BOLOGNA

Main Speakers

Gianluca Faraco (Universitá di Milano Bicocca)

Title: Period realisation of meromorphic differentials

Abstract: Let S be an oriented surface of genus g and n punctures. The periods of any meromorphic differential on S, with respect to a choice of complex structure, determine a representation $\chi : \Gamma_{g,n} \to \mathbb{C}$, where $\Gamma_{g,n}$ denotes the first homology group of S. Chenakkod-F.-Gupta characterised the representations that thus arise, that is, lie in the image of the period map $\mathsf{Per} : \Omega\mathcal{M}_{g,n} \to \mathsf{Hom}(\Gamma_{g,n}, \mathbb{C})$. This generalises a classical result of Haupt in the holomorphic case. Moreover, we determine the image of this period map when restricted to any stratum of meromorphic differentials, having prescribed orders of zeros and poles. Strata generally fail to be connected and in fact they may exhibit connected components parametrised by some additional invariants. In collaboration with D. Chen we extend the earlier result by Chenakkod-F.-Gupta to connected components of strata.

Francesco Fournier Facio (ETH Zürich)

Title: Groups *not* acting on hyperbolic spaces

Abstract: Geometric group theory likes to understand groups by the way they act on hyperbolic spaces. By now there is a huge machinery that allows to get a lot out of this philosophy. We will discuss the class groups that are *not* approachable this way, in a precise sense: it turns out that this class is quite well-behaved, and contains many interesting groups, especially of dynamical origin. This is joint work with Sahana Balasubramanya, Anthony Genevois and Alessandro Sisto.

Alessio Savini (Université de Genève)

Title: The mysterious... punctured plane

Abstract: Among the topological spaces we first meet in our studies, we can surely find the punctured plane, namely $\mathbb{R}^2 \setminus \{0\}$. Even if the latter space may appear harmless, we can easily complicate the situation by viewing it as a homogeneous $\mathrm{SL}(2,\mathbb{R})$-space where the stabilizer of a point is (conjugated to) the unipotent radical of the minimal parabolic subgroup corresponding to upper triangular matrices. In this way, we can study the measurable cohomology of the action of $\mathrm{SL}(2,\mathbb{R})$ on $\mathbb{R}^2 \setminus \{0\}$ and we can ask if the usual evaluation map, having as target the measurable cohomology of $\mathbb{SL}(2,\mathbb{R})$, is actually an isomorphism or, at least, it is surjective.

In this talk we will see that surjectivity holds in a very general setting and we will show that a non-trivial kernel appears. We will describe such kernel in the case of the punctured plane by writing an explicit cocycle whose nature remains very mysterious to us. This is a joint work with Michelle Bucher.

Junior Speakers

Federica Bertolotti (SNS Pisa)

Title: Triangulation Complexity and Integral simplicial volume

Abstract: Both the Triangulation Complexity and the Integral Simplicial volume are integral invariants associated with oriented closed topological manifolds and both can be intuitively defined as the measure of the complexity of a manifold in terms of simplices. Formally, given an oriented closed connected manifold M, the triangulation complexity of M is the minimum number of simplices appearing in a triangulation of M, while the integral simplicial volume is the minimum of the ℓ^1-norms of integral fundamental cycles of M. It is easy to see that the two invariants coincide in dimensions 1 and 2; however, already in dimension 3, triangulation complexity and integral simplicial volume can be very different quantities.

During this talk, we will see the definitions of these two objects, try to understand why they coincide in dimensions 1 and 2 and construct an infinite family of 3-manifolds for which the ratio between these two quantities is not bounded.

Viola Giovannini (Université du Luxembourg)

Title: Renormalized volume of convex cocompact hyperbolic 3-manifolds.

Abstract: Given a hyperbolizable 3-manifold N, the renormalized volume is a real smooth function on the space of convex cocompact hyperbolic structures on N, which gives interesting information on the geometry of the manifold. Since the space of convex cocompact hyperbolic structures is parametrized by the Teichmüller space of the boundary, one can look at the gradient flow of the renormalized volume with respect to the Weil-Petersson metric. After a brief introduction to the renormalized volume, we will see how this is related to the volume of the convex core, pointing out the differences between the incompressible and the compressible boundary cases. We will then see some results arising from the study of the gradient flow.

Giovanni Italiano (SNS Pisa)

Title: Hyperbolic manifolds fibering over the circle

Abstract: We provide some examples of hyperbolic 5-manifolds fibering over the circle, showing that this phenomenon is not restricted to dimension 3. One consequence of this result is the existence of a hyperbolic group with a finite-type subgroup that is not hyperbolic. The main tool to build the fibration is Bestvina-Brady Morse theory applied to a hyperbolic n-manifold that decomposes into right-angled polytopes, enriched with a combinatorial game recently introduced by Jankiewicz, Norin and Wise. Joint work with B. Martelli and M. Migliorini.

Bianca Marchionna (Universitá di Milano Bicocca - Bielefeld University)

Title: Trees, buildings and their interactions

Abstract: It is a common approach to study the structure of groups by investigating their actions on geometric and combinatorial objects, e.g., trees or buildings. After a brief introduction of the two structures, we will discuss a criterion for constructing trees from buildings due to F. Haglund and F. Paulin, possibly sketching an alternative proof. Using the above-mentioned construction, one can produce new cohomological and simplicity results for groups acting Weyl transitively on buildings. Joint work with I. Castellano and T. Weigel.

Matteo Migliorini (SNS Pisa)

Title: Smoothing circle-valued Morse functions

Abstract: While smooth Morse functions are the most widely studied among all notions of Morse theory, it is not always easy to define a smooth function, especially when the manifold is defined by some cell decomposition. In this case, it is preferable to define maps separately on every cell, and then glue them together to obtain a map defined on the whole manifold. This carries the problem that is not clear a priori whether the manifold admits a smooth structure to begin with, and whether the map can be smoothened accordingly. In this talk, I will present a piecewise-linear analogue of smooth Morse functions, and try to address the question of when a piecewise-linear circle- valued Morse function can be smoothened while keeping the same number of critical points of the same index.

Francesco Milizia (SNS Pisa)

Title: Simplicial volume and the reflection trick

Abstract: The Davis' reflection group trick is a powerful construction that, on several occasions, has been used to find aspherical manifolds with exotic properties. It takes, as input, a triangulation of the boundary of a compact manifold; if the input satisfies suitable properties, the output is a closed aspherical manifold. The resulting manifolds provide an excellent testing ground for a long-standing conjecture of Gromov, which links the vanishing of the Euler characteristic to that of the simplicial volume (another homotopy invariant of manifolds). The challenge is to understand when the resulting manifold has vanishing, or positive, simplicial volume. In the talk, after an introduction to the problem, I will describe a characterization of the triangulations of the 2-sphere that give rise to manifolds with positive simplicial volume, and then tell what I know about the much more difficult case of the 3-sphere. This is a work in progress.

Diego Santoro (SNS Pisa)

Title: An introduction to the L-space conjecture

Abstract: Heegaard Floer homology was defined by Ozsváth and Szabó in the early 2000's. It consists of a package of invariants of closed oriented 3-manifolds and it has found many important and profound applications in low dimensional topology. In this talk I will introduce the L-space conjecture, that boldly predicts strong connections among properties relating Heegaard Floer homology, foliations and the fundamental group of an irreducible rational homology 3-sphere.

Matteo Tarocchi (Universitá di Pavia)

Title: Thompson-like groups acting on fractals

Abstract: The trio of Thompson groups F, T and V has made its appearance in many different topics, and these groups are so ubiquitous that M. Brin called them "Chameleon groups" in 1996. Introduced in the '60s by Richard Thompson, the groups T and V were the first examples of infinite finitely presented simple groups, whereas the fame of its smaller sibling F mostly originates from a question that has been open for decades and still is: whether it is amenable or not. In 2019 J. Belk and B. Forrest introduced a generalization of Thompson groups, the family of Rearrangement Groups. These are groups of certain "piecewise-canonical" homeomorphisms of fractals that act by permuting the self-similar pieces that make up the fractal. This talk will introduce Thompson groups and Rearrangement Groups, and will highlight some known facts about them.

Enrico Trebeschi (Universitá di Pavia)

Title: Constant mean curvature hypersurfaces in Anti-de Sitter space

Abstract: The *asymptotic* Plateau problem in the hyperbolic $n-$dimensional space consists in finding a minimal (or CMC, namely of constant mean curvature) hypersurface with prescribed boundary at infinity.

In this talk, we will study a similar problem in the Anti-de Sitter space, *i.e.* the Lorentzian analogous of the hyperbolic space, and show that there exists a unique smooth hypersurface having constant mean curvature $H \in \mathbb{R}$ and with a suitable prescribed boundary data.

Furthermore, all the hypersurfaces mentioned above are complete. This result can be seen as the extension of Cheng-Yau theorem to the negative constant sectional curvature case.